

NEW ELECTROWEAK INSTANTON AND POSSIBLE BREAKDOWN OF UNITARITY

F. R. Klinkhamer

Institut für Theoretische Physik, Universität Karlsruhe,
D-76128 Karlsruhe, Germany

Abstract

Potential implications of a new constrained instanton solution in the electroweak standard model are discussed. Notably, there may be a non-perturbative unitarity violating contribution to the total cross-section at high collision energies.

1. Introduction

In the last year we have given for the electroweak standard model :

1. an explicit construction of a new static, but unstable, classical solution, the sphaleron S^* [1] ;
2. an existence “proof” (and construction method) for the related constrained instanton I^* [2] .

Roughly speaking, S^* is a constant time slice through I^* , just like the well-known sphaleron S [3] resembles a constant time slice of the BPST instanton I [4].

The outline of this talk is as follows. First we recall a few pertinent facts about these two new classical solutions. Then we make some remarks on the potential physics implications, focussing on the role of the new instanton I^* . Specifically, we mention the asymptotics of perturbation theory and the apparent violation of unitarity at high energies.

2. Classical solutions

The sphaleron S^* has the following characteristics [1] :

1. axial symmetry of the fields ;
2. vanishing Higgs field at two points on the symmetry axis, separated by a distance d_{S^*} ;
3. chiral fermion zeromodes (related to the global $SU(2)$

anomaly), localized at either point of vanishing Higgs field, depending on the chirality ;

4. energy $E_{S^*} \sim 2 E_S \sim 20 \text{ TeV}$ and $d_{S^*} \sim 4 M_W^{-1}$.

The instanton I^* has not yet been constructed in all detail, but at least the following properties are clear [2], starting with a technical preliminary :

0. constraint needed to fix the scale (ρ) of the solution ;
1. axial symmetry of the fields ($U(1)$ -equivariance) ;
2. Higgs zeros separated by a distance d_{I^*} ;
3. localized chiral fermion zeromodes ;
4. action $A_{I^*} \sim 2 A_I \sim (16 \pi^2 / g^2) (1 + O(\rho^2 M_W^2))$ and distance parameter $d_{I^*} \sim M_W^{-1} (2 + O(\rho M_W))$;
5. resemblance to a very loose di-atomic molecule for scales $\rho \ll M_W^{-1}$.

3. Perturbation theory

The instanton I^* sits at the top of a non-contractible loop of 4-dimensional euclidean configurations and is assumed to have only one negative mode. In a way I^* is like the sphaleron of a 5-dimensional theory. (Note that I^* is not a “bounce” solution [5], because of the absence of a “turning point” with vanishing field derivatives.)

Following Lipatov [6] we see that I^* determines the a-

symptotics of electroweak perturbation theory

$$c_k g^{2k} \propto \frac{k!}{(A_{I^*})^k} \sim \frac{k!}{(16\pi^2)^k} g^{2k}, \quad (1)$$

where c_k are the coefficients calculated for an arbitrary Green's function. The same behaviour has actually been verified for the groundstate energy in a quantum mechanical model [7].

The result (1) on the asymptotics can be rephrased [8] by saying that the solution I^* gives a singularity at $z \sim 16\pi^2$ in the Borel plane (variable z corresponding to g^2).

4. Unitarity

A straightforward calculation [2, 9] of the two-fermion forward elastic scattering (FES) amplitude gives from the euclidean path integral (the dominant contribution being close to $\rho = 0$)

$$F(s, 0)^{\text{non-pert}} \propto \exp \left[\sqrt{s} d_{I^*}(0) - A_{I^*}(0) \right], \quad (2)$$

with $F(s, t)$ the scattering amplitude as a function of the Mandelstam variables. This behaviour follows from inserting the I^* fields into the euclidean path integral for the 4-point Green's function and integrating over the collective coordinates. The first term in the exponent (2) comes from the Fourier transform of the fermion zero-modes, which are asymmetric with a distance parameter d_{I^*} , and the analytic continuation from euclidean to minkowskian space-time. The second term is simply the instanton action.

Remark that our calculation is similar to an earlier one with an *approximate* solution $\bar{I}\bar{I}$ [10, 11]. We use, instead, the only known *exact* solution (I^*) relevant to the problem.

The non-perturbative contribution (2) is generic to all FES amplitudes and violates unitarity at

$$(\sqrt{s})_{\text{threshold}} \sim \frac{A_{I^*}(0)}{d_{I^*}(0)} = \left(\frac{A_{I^*}(0)}{16\pi^2/g^2} \right) \left(\frac{2M_W^{-1}}{d_{I^*}(0)} \right) E_{S^*}, \quad (3)$$

with the definition

$$E_{S^*} \equiv 2\pi M_W / \alpha_w \sim E_{S^*}.$$

The point is that by the optical theorem (unitarity) the imaginary part of the FES amplitude should be related to the total cross-section, with the Froissart bound (unitarity and analyticity) $\sigma^{\text{total}} < O(\log^2 s)$, and this bound is rapidly violated by the exponential increase with center of mass energy \sqrt{s} as given by (2). More directly, the exponential behaviour (2) violates the

polynomial boundedness condition $|F(s, 0)| < s^N$, with N a finite power, see for example [12].

Clearly this is a serious problem for electroweak field theory which *must* be solved. We see three possible solutions :

1. unitarity restoration together with the Feynman perturbation series ;
2. inapplicability of the conventional euclidean path integral formalism, cf. [13] ;
3. modification of the standard model .

Elsewhere we hope to elaborate on the first, most conservative, alternative. Here we only remark that this possible solution may provide us with additional constraints on the parameters of the theory.

If, however, there is a significant B+L violating part to σ^{total} from (2), then solutions 2 and/or 3 may be forced upon us.

5. Conclusions

New classical solutions of the electroweak field equations have been discovered recently, the sphaleron S^* and the instanton I^* . In this talk we have argued that these classical solutions play a role in some of the most fundamental questions of electroweak field theory, viz. the meaning of perturbation theory and unitarity.

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